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DARK FLUID: TOWARDS A UNIFICATION OF EMPIRICAL THEORIES OF GALAXY ROTATION, INFLATION AND DARK ENERGY

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ABSTRACT

Empirical theories of Dark Matter like MOND gravity and of Dark Energy like f(R) gravity were motivated by astronomical data. But could these theories be branches rooted from a more general hence natural framework? Here we propose the natural Lagrangian of such a framework based on simple dimensional analysis and co-variant symmetry requirements, and explore various outcomes in a top-down fashion. Our framework preserves the co-variant formulation of GR, but allows the expanding physical metric be bent by a single new species of Dark Fluid flowing in space-time. Its non-uniform stress tensor and current vector are simply functions of a vector field of variable norm, resembling the 4-vector electromagnetic potential description for the photon fluid, but is dark (e.g., by very early decoupling from the baryon-radiation fluid). The Dark Fluid framework naturally branches into a continuous spectrum of theories with Dark Energy and Dark Matter effects, including the $f(R)$ gravity, TeVeS-like theories, Einstein-Aether and $\nu\Lambda$ theories as limiting cases. When the vector field degenerates into a pure Higgs-like scalar field, we obtain the physics for inflaton and quintessence. In this broad setting we emphasize the non-constant dynamical field behind the cosmological constant effect, and highlight plausible corrections beyond the classical MOND predictions. Choices of parameters can be made to pass BBN, PPN, and causality constraints. The Dark Fluid is inspired to unify/simplify the astronomically successful ingredients of previous constructions: the desired effects of inflaton plus quintessence plus Cold DM particle fields or MOND-like scalar field(s) are shown largely achievable by one vector field only.

Subject headings: Gravitation; Cosmology: theories; Dark Matter; Galaxies: kinematics and dynamics

1. INTRODUCTION

Gravity, the earliest and the weakest of the known forces, has never been very settled. The beauty of co-variant symmetry motivated Einstein to supersede the Newton's paradigm with General Relativity, but (inadequate) empirical evidences motivated Einstein to introduce first and then withdraw the cosmological constant, a concept defying quantum physics understanding even in modern day. While making generally tiny or a factor of two corrections, the equivalence principles insist on certain symmetries in space-time, *e.g.*, co-variant symmetry and no frame to measure locally any absolute direction of gravitational acceleration for a free-falling observer. However, symmetry can be spontaneously broken if there are dynamical interactions or couplings of fields; a well-known mechanism in several branches of physics, especially the Higgs-mechanism in particle physics to give a mass to a particle. Many attempts have been made to break the strong equivalence principle by adding new fields (degrees of freedom) in the gravity sector, which essentially means the gravitational "constant" G may be is a new dynamical degree of freedom governed by other fields coupled to the metric. The best known is the Brans-Dicke theory (1961). The lesser known is that a vector field of non-zero absolute value in vacuum can also be coupled to gravity, to give absolute directions (Will 1993). It has long been suggested that Lorentz symmetry can be broken locally in the quantum gravity and string theory context (Kostelecky & Samuel 1989) to yield a vector field of a non-zero expectation value (*e.g.*, pointing towards the direction of time) in the vacuum. The most successful attempt so far is the Einstein-Aether theory of Jacobson et al. (2001). A common theme of these theories is that they are *not* invented for certain observational anomaly. Rather in the same vein as how symmetry motivated General Relativity, these theories meet the astronomical data only a posteri, e.g., Li, Mota, Barrow (2007) showed a vector field in the gravity sector could NOT be excluded by the accurate Cosmic Microwave Background (CMB) data.

Nevertheless, the above order is not the only way to discover theories. The puzzling black body radiation spectrum and Balmer's curious empirical formula for hydrogen lines are among the odd pieces of classical physics, which lead to full formulation of quantum mechanics. This bottom-up approach is often gradual, the arrival of the final theory taking several generations of formulations (*e.g.*, from Planck's model for blackbody radiation and Bohr's model for hydrogen atom to Heisenberg's matrices-based formulation in general) with different levels of mathematical rigour and sophistication.

Historically, Milgrom's MOdified Newtonian Dynamics (MOND) was invented without any packaging by co-variant theories of gravity, just as the Dark Matter empirical concept was invented by Zwicky without packaging first with SuperSymmetry-like particle field theories. MOND is a formula motivated to reveal the curious uniform rules (or facts) underlying rotation curves of many spiral galaxies, as Balmer's formula and its generalisations suggesting strongly a fundamental rule for all atomic lines. Since the rule is empirical and bare

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(without co-variance), it waits to be enshrouded by a theory preserving basic symmetries to predict any logical corrections to situations where the empirical rule must fail slightly, *e.g.*, by factor of two in some gravitationally lenses made by elliptical galaxies and clusters of galaxies. The Tensor-Vector-Scalar (TeVeS) framework of Bekenstein (2004), unifying earlier constructions by Sanders and others, makes the first step to the integration of MOND formula with fundamental physics. A time-like vector field is shown to be the necessary ingredient of a MOND gravity. Yet the original aim of TeVeS was limited, *e.g.*, not addressing the cosmological constant problem, or the inflation. Orthogonally many literatures considered theories of modified gravity such as the $f(R)$ gravity (Chiba 2003) and scalar inflation theory as *ad hoc* fixes of the cosmological constant problem and the horizon problem respectively, without aiming to address outstanding questions on galaxy rotation curves. Recently Zhao (2007), built on the work of Zlosnik et al. (2007), showed that these outstanding problems of DM and DE can find at least one common solution simultaneously in the framework of a massive vector field. In these theories, there is "One Field which rules them all and in the darkness bind them."

The most famous examples of a vector field is the massless spin-1 photon and the massive Z-boson in the electroweak theory. The standard way to give masses to particles in particle physics is the Higgs mechanism where a scalar field, coupled to the vector field, acquires a nonzero value in vacuum. Turning to the gravity sector, however, it is unclear how the fundamental prediction of spontaneous symmetry breaking from quantum gravity might be related to the mundane effects of galactic dark matter and cosmological constant. We lack a framework.

We shall show that MOND, $f(R)$ gravity, Einstein-Æther theory and inflation can be integrated into a common framework with a unit vector and a dynamical scalar field. MOND would become a specific choice of the potential of the scalar field. Having such a framework allows one to explore the consequences of modified gravity systematically. It can be meaningless to even differentiate dark energy and modified gravity. Modified gravity contains extra fields, which can be treated as dark energy field.

The goal of this paper is to show the existence of a very general Lagrangian for which the MOND formulae are the natural consequences in spiral galaxies in equilibrium, rather than the golden rule for (non)-equilibrium systems of all scales. We demonstrate this with the modified Poisson equation and for the equation for the Hubble expansion.

The outline is as follows. We propose our general Lagrangian in §2, and illustrate how it reduces to various special cases, TeVeS, BSTV, Einstein-Aether, $f(K)$, $f(R)$, inflation. We choose a subset of models with MOND and Dark Energy effects in §3. We examplify the properties of our dark fluid in the case of Hubble expansion and inflation (§4), and for static galaxies (§5). We discuss corrections to MOND in §6, and summarize the properties of the Dark Fluid in §7. Appendix gives an estimate of the damping frequency of the Dark Fluid.

2. THE PROPOSED LAGRANGIAN FOR THE DARK FLUID

Denote a vector field by Z^a, which has generally a variable or dynamic norm

$$\varphi^2 \equiv g_{ab} Z^a Z^b, \quad (1)$$

which is essentially an auxiliary scalar field characterizing the norm of the vector field Z^a, hence is not an independent dynamical freedom. A generic coupling of the vector field Z^a with the space-time is through the contractions among the $Z^a Z^b$ tensor, the g_{ab} metric tensor, the Ricci tensor R_{ab}. Hence the most generic theory of the vector should contain the terms

$$L = \left[g^{ab} + g^{ab} f_1 \varphi^2 - f_2 Z^a Z^b\right] R_{ab} + f_{34}, \quad (2)$$

$$f_{34} \equiv f_3 \varphi^2 + f_4 \nabla^a \varphi \nabla_a \varphi + \cdots \quad (3)$$

where f_i could be constants or functions of φ. The term $f_2 Z^a Z^b R_{ab} = f_2 Z^a \left(-\nabla_a \nabla_c Z^c + \nabla_c \nabla_a Z^c\right)$ could always be recast through a full divergence to $\to K^{ab}_{cd} \nabla_a Z^b \nabla_c Z^d$ where K^{ab}_{cd} are constant tensors or tensor functions. The equivalent expression for the Lagrangian is

$$L = K^{ab}_{cd} \nabla_a Z^b \nabla_c Z^d + \Theta_0 R + \Theta_1 \varphi^2 + \Theta_2 \nabla^a \varphi \nabla_a \varphi + \cdot (4)$$

where $R = g^{ab} R_{ab}$ is the Ricci scalar, and K^{ab}_{cd} are constant tensors for simplicity but can also be lengthy functions of φ in general; Θ_i are constants in the simplest case but functions of φ in general.

The above Lagrangian is generic enough, and many dark energy models can be derived from it. For example, the terms $(1+\varphi^2)R + \omega \varphi^{2/(1+n)}$ can lead to an $R + \mathrm{const}/R^n$ (Li & Barrow 2007) gravity (as could be checked by solving φ from the equation of motion of φ and then substituting back into the original Lagrangian); and the terms $\phi R - 4(1-\phi^{-1})\omega \nabla^a \varphi \nabla_a \varphi$ leads to the Brans-Dicke theory of gravity, where the auxilliary field $\phi = 1 + \varphi^2$. The usual inflation-like or quintessence theories can be recovered from the terms $R + V(\varphi) + \nabla^a \varphi \nabla_a \varphi$.

The essential dynamics of a cosmological vector field is described by the first term $K^{ab}_{cd} \nabla_a Z^b \nabla_c Z^d$ (Ferreira et al. 2007, Halle, Zhao & Li 2008). If insisting that the field has a unit norm guaranteed by a Lagrange multiplier, one would recover Einstein-Æther theory (Jacobson et al. 2001) and its generalizations (Zlosnik *et al.* 2007, Li *et al.* 2007).

3. MOND-INSPIRED SUBSET OF DARK FLUID THEORIES

To see the relation to MOND, consider the subset where $\Theta_0 = 1$, $\Theta_1 \sim \Lambda_0$, and $\Theta_2 \sim N^{-2}$. Let's decompose the four dynamical freedoms in the vector field into a unit norm part of 3 degrees of dynamical freedom,

$$Æ^a \equiv \frac{Z^a}{\varphi}, \quad \lambda \equiv \varphi^2 \equiv g_{ab} Z^a Z^b, \quad (5)$$

and rewrite the scalar field φ in term of the new scalar field λ. The Lagrangian is then casted to a form containing *at least* the following

$$\mathcal{L} = L_m + R + L_\varphi + L_Æ \quad (6)$$

$$L_\varphi = -\frac{1-c_\varphi^2}{N^2} \nabla_\parallel \varphi \nabla_\parallel \varphi - \frac{c_\varphi^2}{N^2} \left(\nabla_a \varphi\right)\left(\nabla^a \varphi\right) + 2\Lambda_0 F(\lambda) + \cdot (7)$$

$$L_Æ = c_4 \nabla_\parallel Æ_c \nabla_\parallel Æ^c + c_2 \left(\nabla_a Æ^a\right)^2 + \left(Æ^a Æ_a - 1\right) L^* + \cdots, (8)$$

where N, F, c_φ^2, c_2 and c_4 are various coupling constants or functions of $\lambda \equiv \varphi^2$, and $\nabla_\parallel \equiv Æ^a \nabla_a$, $\nabla^a = g^{ac} \nabla_c$,

$Æ_a = g_{ac}Æ^c$, R is the Ricci scalar, and L^* is the Lagrange multiplier (a kind of potential). The Lagrangians L_m, L_φ, $L_Æ$ are for the matter, the scalar field φ and the unit vector field $Æ^a$ respectively, where we omitted two possible terms $c_1K_1 + c_3K_3 = c_1(\nabla_a Æ_b)(\nabla^a Æ^b) + c_3(\nabla_a Æ_b)(\nabla^b Æ^a)$ in $L_Æ$. Λ_0 is the only dimensional scale in the dark fluid, it is a scale of energy density.

In this Lagrangian we have three dynamical fields: the scalar field λ, the Æther field $Æ^a$ and the metric field g^{ab} plus a non-dynamical L^*. Now varying the action $S = -\int \sqrt{-g}d^4x\frac{L}{16\pi G}$ with respect to them will lead to the scalar field equation of motion (EOM), Æther field EOM and the modified Energy-momentum tensor plus the unit vector constraints for Æ field respectively. The general results are more tedious and are presented elsewhere (Halle, Zhao, Li 2008). Here we illustrate the physics by considering only the main terms for a specific choice of functions.

3.1. *Choices of coupling constants*

The dimensionless function $F(\lambda)$ has the meaning of the potential of the scalar field $\lambda \equiv \varphi^2$, and c_φ^2, c_2 and c_4 are of order unity, and are also generally functions of λ and will be shown to be related the sound speeds of the Dark Fluid.

- We set the scalar field sound speed
$$c_\varphi \sim 1, \tag{9}$$
and we shall treat N and c_φ as constants.

- We choose the coefficients
$$\frac{c_4(\lambda)}{2} = \lambda \equiv \varphi^2 \tag{10}$$
$$\frac{c_2(\lambda)}{2} \equiv \frac{b(\lambda)-1}{3} = -\frac{1}{3\lambda}, \tag{11}$$
i.e. $b = 1 - \varphi^{-2}$. For simplicity we set two other terms in $L_Æ$ of Jacobson's unit vector field to zero, i.e., $c_1K_1 = 0$ and $c_3K_3 = 0$. This might not be necessary, but simplifies the analysis of PPN parameters in the solar system and the sound speed of the vector field. Our choice of Lagrangian with $c_1 = c_3 = 0$ kills spin-1 mode waves of the vector field, and gurantees that the normal gravitational wave in the tensor mode will propagate with the normal speed (of light). This choice is perhaps not necessary, but is intended to avoid contraversy on the causuality issue. Even the spin-0 mode sound speed $c_Æ$ is plausible: a rigourous analysis (Foster & Jacobson 2006, valid for any constant λ and b) predicts
$$c_Æ^2 = \frac{c_2}{c_4}\frac{(2-c_1)}{(2+3c_2)} = \frac{(1-b)(\lambda-1)}{3b\lambda} = \frac{1}{3\varphi^2} > 0. \tag{12}$$
Interestingly in the solar system, where $\lambda \equiv \varphi^2 \to 0$, the spin-0 mode of the vector field propagates almost instantaneously with $c_Æ$ being $(3\lambda)^{-1/2}$ times bigger than the speed of light, avoiding the Cherenkov radiations constraint in the solar system. All PPN parameters are expect to be equal to that of GR in the solar system as well; although the PPN parameters $\alpha_1 = -8\lambda$, $\alpha_2 = (3\lambda - 1)\lambda$ are non-zero, as shown later the scalar field λ is expected to settle to a very small equilibrium value $\sim (10^{-10})^4$ on earth for our choice of the penalizing scalar field potential $F(\lambda)$.

- Our choice for the dark fluid potential (shown in Fig.1) is
$$\Lambda_0 F(\lambda)|_{\lambda=\varphi^2} = \frac{8(\varphi-1)^3}{3}\Lambda_0. \tag{13}$$
The dark fluid's energy scale is defined by Λ_0. An important property is that in the limit $\lambda = \varphi^2 \to 1$, we have
$$F' \equiv \frac{d}{d\lambda}F \sim (1-\lambda)^2, \text{ if } \lambda = \varphi^2 \to 1, \tag{14}$$
$$\propto \lambda^{-1/2}, \text{ if } \lambda = \varphi^2 \to 0; \tag{15}$$
note a prime always means $\frac{d}{d\lambda}$). We shall show that this property describes *a non-uniform (dark energy) fluid which gives the MOND-like (dark matter) effects in galaxies but not in the solar system.* [3]

4. BACKGROUND COSMOLOGY

Consider background cosmology in the FRW flat metric,
$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2). \tag{16}$$
Firstly, the scalar field follows an equation of motion exactly as a quintessence,
$$\ddot{\varphi} + 3H\dot{\varphi} = -\left(\Lambda_0 F' + 3b'H^2\right)(2N^2\varphi), \tag{17}$$
so φ, the norm of the vector field Z^a, tracks the Hubble rate $H = \dot{a}/a$. The vector field equation of motion gives the Lagrange multiplier (or the mass of the vector field) $L^* = \partial_t(\alpha H) + \frac{1-c_\varphi^2}{N^2}\dot{\varphi}^2$, where $\alpha = 2b - 2$. This mass is varying with time, hence the vector field describes effectively an unstable slowly decaying particle.

The modified 00-term of the Einstein equation becomes
$$3H^2 = 8\pi G\left[\bar{\rho} + \bar{\rho}_{DM} + \bar{\rho}_{DE}\right], \tag{18}$$
$$\bar{\rho}_{DM} \equiv (b^{-1} - 1)\bar{\rho} \tag{19}$$
$$\bar{\rho}_{DE} \equiv \frac{1}{8\pi G b}\left[\frac{1}{2N^2}\dot{\varphi}^2 + \Lambda_0 F\right], \tag{20}$$
where $\bar{\rho}$ is the (background) energy density of baryon-radiation fluid. For our choice of b,
$$b^{-1} = \frac{\varphi^2}{\varphi^2 - 1}, \tag{21}$$
so we get a dark-matter-like effect of the vector field for $1 < \lambda = \varphi^2 \leq 2$ by amplifying the gravitational constant G by a factor $\infty > b^{-1} \geq 2$. There is no dark matter-like effect at very high redshift, e.g., radiation era or BBN, where $\lambda \to \infty$, hence $b \to 1$. So the BBN constraint is automatically satisfied because the Hubble expansion rate at BBN is equal to that of a radiation only universe.

[3] A more general choice of dark fluid potential of this property is $F \propto \int (\varphi^n - 1)^2 d\varphi$ for $n = 1$ (as above) and $n = 2, 3, 4, \ldots$. These potentials are always simple polynomial functions of φ.

Equivalently the Einstein equation can be written as

$$-\left(2\frac{\ddot{a}}{a}+H^2\right)=8\pi G\left(\bar{p}+\bar{p}_{DM}+\bar{p}_{DE}\right) \quad (22)$$

$$\bar{p}_{DM}=(b^{-1}-1)\bar{p} \quad (23)$$

$$\bar{p}_{DE}\equiv\frac{1}{8\pi Gb}\left[\frac{1}{2N^2}\dot{\varphi}^2-\Lambda_0 F+4b'H\varphi\dot{\varphi}\right] \quad (24)$$

where the pressure of the baryon-radiation fluid $\bar{p}=0$ in the matter-dominated era, hence $\bar{p}_{DM}=0$. Apply the slow-roll approximation in the late universe we find the effective pressure of the vector field

$$-\bar{p}_{DE}\sim\rho_{DE}\sim\frac{\Lambda_0}{8\pi Gb}F\sim\frac{\Lambda_0}{3\pi G}\frac{(\varphi-1)^2\varphi^2}{(\varphi+1)}. \quad (25)$$

This behaves like a dark energy with

$$w=\bar{p}_{DE}/\bar{\rho}_{DE}\sim-1 \quad (26)$$

and with a characteristic scale Λ_0, which must be set of order $(8\times10^{-10}m/s^2)^2$ to match the observed cosmological constant. Note that in writing above equations we have implicitly assumed that the effective dark matter and effective dark energy components couple to each other. This can been seen by checking that neither $\bar{\rho}_{DM}$ nor $\bar{\rho}_{DE}$ satisfies the conservation law $\dot{\bar{\rho}}+3H(\bar{\rho}+\bar{p})=0$, but their sum does. The coupling strength is determined by b': if b is a constant (equals to unity for our model in the early universe), then the two components decouple.

Models with a very large N could even drive inflation in the early universe when the energy density is dominated by the norm of the vector field (*i.e.*, the scalar field φ). It is easy to verify the solution

$$\Lambda_0\frac{8\varphi^3}{3}\sim3H^2b,\quad\frac{d\varphi^2}{d\ln a}\sim-6N^2b^{-1}, \quad (27)$$

applies in the slow-roll phase, where $b\sim1$ for very large φ. This phase of slow rolling can inflate the universe by a factor $\sim\exp\left(\frac{\varphi_i^2-\varphi_f^2}{6N^2}\right)$ (Kanno & Soda 2006). For the universe to inflate by a factor exp(60), *e.g.*, φ rolles from an initial value $\varphi_i\sim20N$ to a final value $\varphi_f\sim10N$. The end of inflation or the start of the radiation era is at a time $H^{-1}\sim\left(\frac{\Lambda_0 F}{3}\right)^{-1/2}\sim10^{17}F^{-1/2}\text{sec}\sim10^5\left(\frac{N}{10^8}\right)^{-3/2}$ sec; the time scale can be even shorter for other forms of Dark Fluid potential [4] The inflation might end when the vector field decays partly into known particles via some small coupling perhaps of the type $g_{ab}Z^aJ^b$ between the vector field and the current field J^b of some known fields, e.g., coupling with sterile neutrinos, which would then mix to neutrinos of all flavors and couple to photons, leptons and hardrons etc.

5. STATIC GALAXY LIMIT

To work out perturbations in static galaxies, remember that in the Newtonian gauge we have only two scalar mode perturbation potentials, Φ and Ψ, which appear in the perturbed metric:

$$ds^2=(1+2\Phi)dt^2-a_0^2(1+2\Psi)(dx^2+dy^2+dz^2) \quad (28)$$

[4] e.g., if $F\propto\int(\varphi^4-1)^2d\varphi\propto\varphi^9$ then $H^{-1}\sim10^{-19}\left(\frac{N}{10^8}\right)^{-9/2}$ sec.

we will let $a_0=1$. We assume NO Hubble expansion.

The vector field equation of motion in static systems fixes $Æ^a=(1-\Phi,0,0,0)$, so the vector field tracks the metric exactly without any freedom in static galaxies.

The 00-component of the Einstein equation becomes a Poisson equation

$$\sum_{i=x,y,z}-(2\Psi)_{,ii}=8\pi G(\rho+\rho_{DM}+b\rho_{DE}), \quad (29)$$

$$\rho_{DM}\equiv\frac{\sum_{i=x,y,z}}{8\pi G}\left[2\lambda\Phi_{,i}\right]_{,i} \quad (30)$$

where we use notation $F_{,i}\equiv\partial_i F$, and the dummy index implies co-variant or contra-variant derivatives with respect to x,y,z. We use the approximation that the DE part $8\pi Gb\rho_{DE}=\frac{\dot{\varphi}^2}{2}+\Lambda_0 F$ is a negligible source compared to $8\pi G\rho$ from the baryons, and that

$$-\Psi=\Phi \quad (31)$$

from the spatial cross term of the Einstein equation. The above result is essentially a Poisson equation where the vector field creates an effective dark matter-like source term ρ_{DM}. Rearrange the terms, the same equation becomes the MOND Poisson equation

$$\nabla\cdot[(1-\lambda)\nabla\Phi]=4\pi G\rho,\quad\lambda\equiv\varphi^2. \quad (32)$$

To see that $1-\lambda$ can be identified with the MOND μ_M function, first we define a value of the scalar field φ_M such that

$$F'|_{\lambda=\varphi_M^2}\equiv\frac{|\nabla\Phi|^2}{\Lambda_0}. \quad (33)$$

We find that the scalar field equation of motion is given as

$$-c_\varphi^2\nabla^2\varphi=-\left[\Lambda_0F'-|\nabla\Phi|^2\right](2N^2\varphi), \quad (34)$$

where we neglect all time-dependent terms. This equation is similiar to the equation of Yukawa potential with a screening length of L, $(\nabla^2-L^{-2})\varphi=0$. In the simplest case, we adopt $c_\varphi^2\to0$ to kill the Laplacian term $\nabla^2\equiv\sum_{i=x,y,z}\partial_i\partial_i$. In the static limit, we find the equation for the scalar field becomes

$$4\Lambda_0(\lambda^{-1/4}-\lambda^{1/4})^2=|\nabla\Phi|^2 \quad (35)$$

for our choice of $F(\lambda)$. The equation can then be solved as

$$\lambda=\varphi^2\to\varphi_M^2=\left(\sqrt{1+\frac{x^2}{16}}+\frac{x}{4}\right)^{-4}|_{x=\frac{|\nabla\Phi|}{\sqrt{\Lambda_0}}}. \quad (36)$$

To see we recover the properties of MOND function μ_M (see $1-\varphi_M^2$ vs x shown in Fig.1), we rewrite the solution of the scalar field as

$$1-\varphi_M^2\equiv\mu_M=\begin{cases}x, & \text{where } x\equiv\frac{|\nabla\Phi|}{\sqrt{\Lambda_0}}\ll1\\ 1-\left(\frac{x}{2}\right)^{-4}, & \text{where } |\nabla\Phi|\gg\sqrt{\Lambda_0}\end{cases} \quad (37)$$

This is exactly the physics of MOND if

$$\sqrt{\Lambda_0}\to a_0 \quad (38)$$

is identified with the MOND acceleration scale a_0. In the solar system or strong gravity regime, the modification factor $1-(x/4)^{-4}\sim1$ to the Newtonian Poisson

equation is small and reduces sharply. In weak gravity, applying spherical approximation around a dwarf galaxy of mass m_b, we have $|\nabla\Phi|^2/a_0 = Gm_b r^{-2}$, and the rotation curve $V_{cir}^2(r) = r\nabla\Phi$. The big success of MOND in dwarf spiral galaxies is to explain their Tully-Fisher relation $V_{cir}^4(r)/(Gm_b) = a_0 \sim 10^{-10}m/s^2$ if $\Lambda_0 \sim (1\times 10^{-10}m/s^2)^2$, which is of the order of magnitude of the observed amplitude of "the cosmological constant" effect. In the intermediate regime, our μ_M resembles the "standard" $\mu = \frac{x}{\sqrt{1+x^2}}$ function of MOND, so it will fit rotation curves of galaxies very well.

6. TEMPORAL AND SPATIAL CORRECTIONS TO MOND: OSCILLATIONS AND DIFFUSIONS

When considering merging systems like galaxy clusters, time-dependent terms $\ddot\lambda \sim \dot\lambda^2 \sim O(\omega^2)$ are important, where $\omega = O(|\mathbf{k}|\sigma)$ is the inverse of the timescale to cross a system of size $|\mathbf{k}|^{-1}$ by stars of velocity dispersion σ in unit of the speed of light. There can also be diffusion on small scale due to a pressure-like term $\nabla^2\lambda = -|\mathbf{k}|^2\lambda$.

The scalar field equation of motion becomes

$$\left[\partial_t^2 - c_\varphi^2\nabla^2 + (1-c_\varphi^2)\eta\partial_t\right]\varphi = -2N^2\varphi\Lambda_0\left[F'(\varphi^2) - F'(\varphi_M^2)\right] \quad (39)$$

$$\sim -(\varphi-\varphi_M)\,\nu^2,\ \nu^2 \equiv 4N^2\Lambda_0 F''(\varphi_M^2) \quad (40)$$

where η^{-1} is a damping time scale due to coupling of φ with the Æ field (cf. Appendix), and the diffusion term $-c_\varphi^2\nabla^2 = c_\varphi^2|\mathbf{k}|^2$ can be neglected if $c_\varphi^2 = 0$. The scalar field φ then follows the equation of a damped harmonic oscillator with a damping rate $(1-c_\varphi^2)\eta$ and a slightly non-linear restoring force $\sim -\nu^2\varphi$ and an external force $\sim \nu^2\varphi_M \sim |\nabla\Phi|^2(2N^2\varphi_M)$. Assuming that the correction due to Hubble expansion [5] is negligible for a very small b', the scalar field φ eventually approaches the MOND-like static solution φ_M, thanks to the damping term with a timescale η^{-1}, which kills any history dependence. Rapid oscillations will likely keep the fluid's time-averaged property close to MOND-like solution as well.

We estimate the oscillation time scale

$$\sqrt{\frac{\varphi}{\ddot\varphi}} \sim \nu^{-1} = (2N)^{-1}(\Lambda_0 F''\lambda)^{-1/2}|_{\lambda=\varphi_M^2} \sim \frac{10^8}{N}\cdot 300\mathrm{yr}, \quad (41)$$

which is about 10^9 years if $N \sim 10$. Here we assume $\varphi_M = \sqrt{\lambda} = O(1) = F''$ for systems of mild gravity ($\sim 10^{-8}$cm/s^2, e.g., clusters; for systems of stronger gravity, the time scale is perhaps longer). In the process of damping there will be a correction to MOND μ_M function by the q term, heuristically, $1-\varphi^2 = 1-\varphi_M^2$ if

$$x \to \sqrt{\frac{|\nabla\Phi|^2}{\Lambda_0} + \frac{q}{2N^2\Lambda_0}}, \quad (42)$$

where $q \equiv \left[\partial_t^2 - c_\varphi^2\nabla^2 + (1-c_\varphi^2)\eta\partial_t\right]$ is an operator. In tidally acting systems the value for φ will oscillate between its pre-merging value and its equilibrium value.

Models with a small N would not give MOND. E.g., if $N = 1-10$, $\pi\nu^{-1} \sim (100-10)$ Gyrs, then the universe would be too young dynamically to have a precise MOND effect in galaxies because φ would not have enough time to respond to the formation of galaxies. Rather φ would lack behind, might remain close to its cosmological average:

$$\varphi \sim \bar\varphi, \quad (43)$$

which would mean a boost of the gravity of the baryon by a constant factor $(1-\bar\varphi^2)^{-1}$ everywhere.

7. GENERIC PROPERTIES OF DARK FLUID

It is still uncertain whether the time-dependent correction and a possible diffusion term are enough to help MOND to explain the Bullet Clusters (Angus et al. 2007, Angus & McGaugh 2008). However, it seems robust that the *Dark Fluid* – described by the field $Z^a = Æ^a\varphi(\lambda)$ – is generally out of phase from the baryonic fluid. There are two types of deviations from MOND in general:

- The Dark Fluid has a natural oscillation on time scales of ν^{-1}, which can be damped on a crossing timescale unless the external forcing is in resonance. A very fast damping would mean an almost instantaneous relation between gravity and the scalar field $1-\varphi^2$, as the μ_M in classical Bekenstein-Milgrom (1984) modified gravity interpretation of MOND. A slow damping would mean a history dependent relation, reminiscent of Milgrom's modified inertia interpretation of MOND: the dark fluid adds a dynamically-varying ineria around the baryons which it surrounds. A possible test could be in galaxies with rotating bar(s), where there could be a phase lag between the bar and the effective Dark Matter (Debattista & Sellwood 1998). This has intriguing consequences to the bar's pattern speed because of non-trivial corrections to the MOND pictures of dynamical friction (Ciotti & Binney 2004, Nipoti et al. 2008, Tiret & Combes 2008); the properties of the dark fluid is in bewteen that of real particle dark halo and that naively expected from MOND.

- The Dark Fluid has a pressure, controled by a propagation speed c_φ, where the speed of light is unity here, and the Dark Fluid can be made *Cold* by $c_\varphi^2 \sim 0$, or *Hot* by $c_\varphi^2 \sim 1$, or *Superluminal* by $c_\varphi^2 \geq 1$. The φ would no longer be a function of the local gravity at $\mathbf{r}$ (as in MOND), rather it is a weighted average of a volume of all points $\mathbf{r}_1$ by a Yukawa-type screening function $\exp(-\frac{\nu|\mathbf{r}_1-\mathbf{r}|}{c_\varphi})$, where

$$\text{Screening Length} = c_\varphi\nu^{-1} \sim c_\varphi\frac{10^8}{N}\times\ 300\text{light years}. \quad (44)$$

Note this spatial correction to MOND can exist even in static systems; even a small pressure term with $c_\varphi^2 \neq 0$ might smooth out MOND effects on small scale structures (wide binaries, star clusters, dwarf galaxies), where the wavenumber $|\mathbf{k}|^2$ is much bigger than in galaxy clusters. The screening length can be set at ~ 100 pc for either a $N \sim 10^8$, $c_\varphi \sim 3\times 10^5$ km/s Hot Dark Fluid *or* a $N \sim 10^4$ and $c_\varphi \sim 30$ km/s Cold Dark Fluid. This scale 100pc is a scale dividing dense star clusters and fluffy dwarf galaxies. Observationally dark matter effects are only seen in the universe on scales

[5] Considering the expansion of the universe would introduce a correction term Jb' in the force, where $J = (3H^2 + 2H\eta)$.

larger than 100pc. It has been challenging for MOND to explain this observed scale (Zhao 2005, Sanchez-Salcedo & Hernandez 2007, Baumgardt et al. 2005).

In conclusion, we find a framework of Dark Fluid theories where MOND corresponds a special choice of potentials or mass for the vector field. The Dark Fluid can run Cold or Hot depending on the sound speed c_φ (which could even be a running function of the vector field). These theories degenerate into scalar field theories for Dark Energy effects in the Hubble expansion. It is possible to create an exact $w=-1$ Dark Energy effect (and a Dark Matter effect for $b^{-1} = \frac{\varphi^2}{\varphi^2-1} > 1$). The scale $a_0 = \sqrt{\Lambda_0}$ in MOND in equilibrium spiral galaxies derives its physics from the amplitude of the dark energy Λ_0. MOND or Dark Matter effects are hence indications of a non-uniform Dark Energy fluid described generally by a vector field Z^a. For non-equilibrium systems like the Bullet Clusters or galaxies with satellites, the properties of the Dark Fluid do not follow exactly the usual expectations of MOND or Cold/Hot Dark Matter, but (not so surprisingly) in between.

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APPENDIX

APPENDIX: THE VECTOR FIELD EQUATION AND THE DAMPING RATE η

Consider the vector field perturbation $æ_j = Y_{,j} \ll 1$ in the compressional spin-0 mode with a potential Y. Define $C_2 = \frac{c_2}{c_4} = \frac{b-1}{3\lambda}$, we note the vector field EOM in the raw form is

$$\partial_t\left[2\lambda(\dot{Y}_{,i}+A_i)\right] - \partial_i\left[2C_2\lambda\left(Y_{,jj}+\theta\right)\right] = -\frac{n}{N^2}\dot{\varphi}\varphi\left[\frac{\varphi_{,i}}{\varphi} - \frac{\dot{\varphi}}{\varphi}Y_{,i}\right] \tag{A1}$$

for the index $i = x, y, z$, where $n = 1 - c_\varphi^2$, and $A_i \equiv u^a\nabla_a u_i$ and $\theta \equiv \nabla_a u^a$, and u^a is the unit four-velocity vector locally. Apply the approximation λ is a very small constant (strong gravity), so that $A_i \sim -\Phi_{,i}$ and $\partial_i\theta \sim -3\dot{\Psi}_{,i}$, and neglect the term $\frac{\dot{\varphi}}{\varphi}Y_{,i}$ because $|Y_{,i}| = |æ_i| \ll 1$ for perturbations, and $|\dot{\varphi}| \ll |\dot{\varphi}_{,i}|$ inside the causual horizon. Replace $\varphi = \sqrt{\lambda}$ and apply ∂_i to both sides, we get

$$\partial_i\partial_t\left[2\lambda(\dot{Y}_{,i}-\Phi_{,i})\right] - \partial_i\partial_i\left[2C_2\lambda(Y_{,jj}-3\dot{\Psi})\right] = -\partial_i\left[\frac{n\dot{\lambda}}{4N^2\lambda}\lambda_{,i}\right] \sim O\left(\frac{n}{4N^2}\right)|\mathbf{k}|^2\omega\lambda. \tag{A2}$$

Furthermore, we neglect the spatial and temporal variations of λ and C_2, factor out 2λ, and define

$$\eta \equiv \sum_{j=x,y,z} æ_j^{;j} = -\nabla^2 Y = |\mathbf{k}|^2 Y, \tag{A3}$$

then the approximate equation for η is obtained:

$$\left[\partial_t\partial_t - C_2\nabla^2\right]\eta \sim -\nabla^2\dot{\Phi} + 3C_2\nabla^2\dot{\Psi} + O(\frac{n\omega}{4N^2})|\mathbf{k}|^2 \sim O(\omega^3)\left[1 + \frac{n}{4}O\left([N\sigma]^{-2}\right)\right], \tag{A4}$$

where $n \equiv 1 - c_\varphi^2$ and C_2 plays the role of sound speed squared. Replace $\Psi = -\Phi$, and replace the partial derivative ∂_i with the wave vector $\mathbf{k}$, and ∂_t with the orbital frequency ω, we get

$$\eta \sim \frac{(1+3C_2)|\mathbf{k}|^2\dot{\Phi}}{-\omega^2 + C_2|\mathbf{k}|^2} \sim \frac{1+3C_2}{1-C_2\sigma^{-2}} \times \omega \sim O([\text{orbit crossing time}]^{-1}). \tag{A5}$$

This estimation is good for fairly hot system $\sigma \gg 1/N \sim 30$km/s if $N \sim 10^4$, and $\sigma \gg \sqrt{|C_2|}$. This is an over-estimation if the vector field has a large (relativistic) sound speed due to a not-so-small $\sqrt{|C_2|}$; this is an under-estimation if the vector field has a small sound speed $\sqrt{|C_2|}$ in resonance with the stellar velocity σ or if the system is very cold $\sigma \leq 1/N$. There will also be corrections of order H/ω in an expanding universe.

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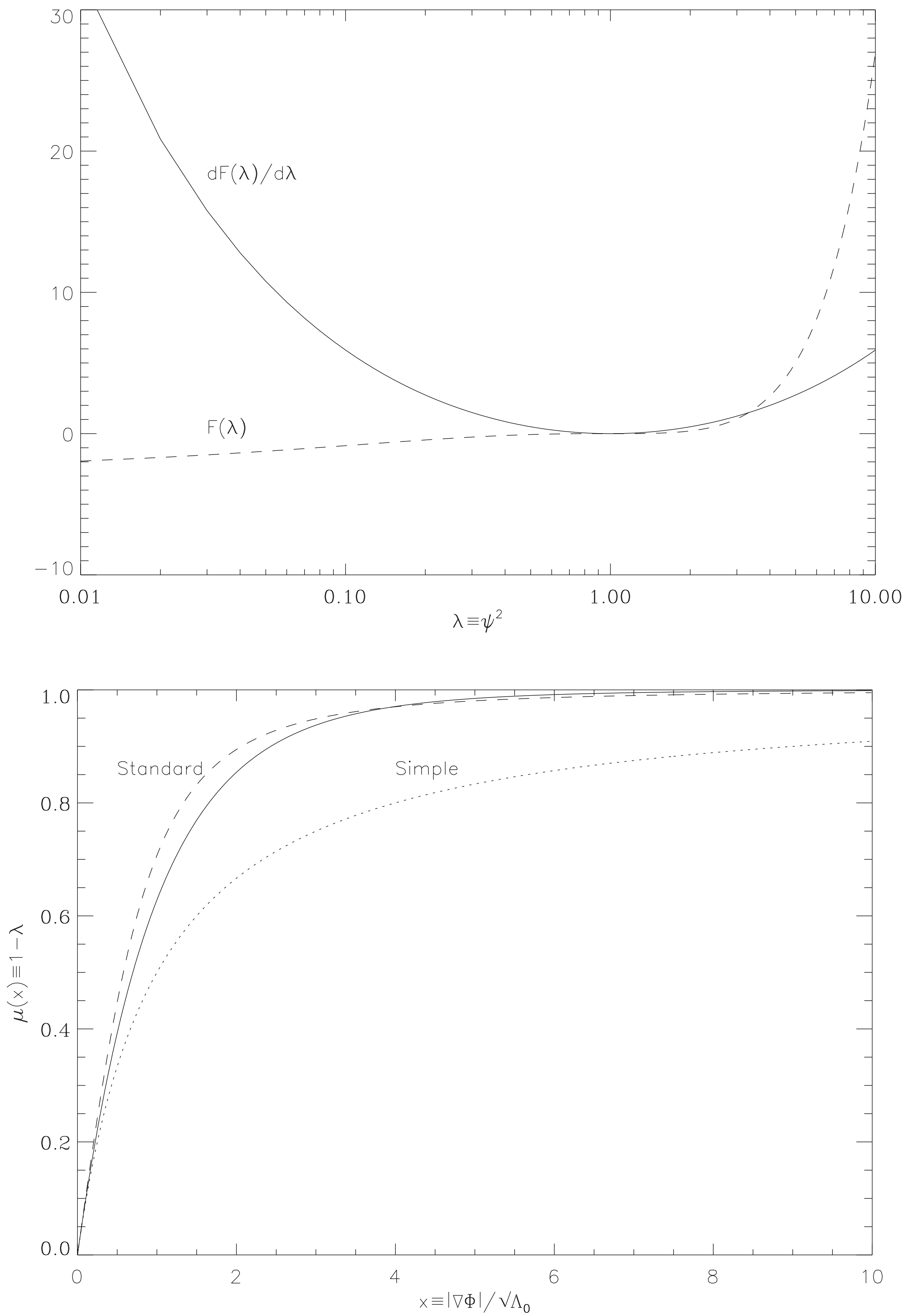

FIG. 1.— Panel (a) shows $F(\lambda) = 8(\varphi-1)^3/3$ (dashed) and $\frac{d}{d\lambda}F = 4(\varphi-1)^2\varphi^{-1}$ (solid) as functions of $\lambda \equiv \varphi^2$. Panel (b) shows our function $\mu \equiv 1-\varphi_M^2 = 1-\left(\sqrt{1+\frac{x^2}{16}}+\frac{x}{4}\right)^{-4}$ as function of $x \equiv |\nabla\Phi|/\sqrt{\Lambda_0}$ (solid). Overplotted is the MOND $\mu = x/\sqrt{1+x^2}$ (dashed, labeled "standard") and the MOND $\mu = x/(1+x)$ (dashed, labeled "simple"), adopting $\sqrt{\Lambda_0} \to a_0$.

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